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RESEARCH ARTICLE

SPECIFIC FEATURES OF RESEARCH AND DEVELOPMENT OF THE PASSIVE REDUNDANT SUBSYSTEMS OF THE AIRCRAFT WITH DUE CONSIDERATION OF TOLERANCES

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ABSTRACT

The goal of this paper is to determine the specific features, which must be taken into consideration in the course of research and development of the passive redundant subsystems with due consideration of tolerances in the cases of sudden independent failures. It was demonstrated that this kind of reservation (this term is also referred to as "redundancy") must take into account presence of many various factors (which exist within the same subsystems): specified tolerances and realizable tolerances; n-tuple redundancy and aliquant redundancy; scales of the tolerances, which have various levels of significance; ranges of probabilities for the components, where redundancy is useful and where it is impracticable one, etc.

KEYWORDS

Sudden failures, passive redundancy, tolerances of the output parameters, degree of redundancy, critical probabilities, failure-free operation.

1. INTRODUCTION

The issues concerning increase of the failure-free operation of various subsystems of the aeronautical equipment, as well as the space and missile equipment in the cases of sudden independent failures determine one of the very important directions of researches. Passive or (as the phrase goes) standby redundancy is often the single method of increase of the failure-free operation of such subsystems. It is connected with inclusion of the redundant components (reserved components) to various subsystems. Such method of redundancy is used for those subsystems, which do not allow even the short-term work stoppages. In these cases, measure of redundancy is estimated with the help of the redundancy degree. In the course of work, the redundancy degree K is understood as the ratio of the total quantity of components of the redundant subsystem "n" to the quantity of the main components "m".

$$K = n/m \tag{1}$$

One of the features of the passive redundancy is impossibility to determine which components are the main ones, and which components are the redundant ones, because of all these components are the same and they operate in the same conditions. It is only possible to determine their quantity.

In the case of passive redundancy of subsystems without due consideration of tolerances, the redundant subsystem is in working order, provided that it has at least one component that functions properly [1-3]. Therefore, the passively redundant subsystem, which consists of the homogeneous components, is in working order, if it has at least one component that functions properly, that is, m = 1. However, the above-referenced statement is only valid in the absence of influence of the output characteristics of the redundant subsystems upon the performance capabilities of the subsystems that are connected with the redundant subsystems. In the presence of the above-referenced influence, it is necessary to take into consideration the tolerance in divergence of the output parameters of the passive redundant subsystems in the cases of failures of those components that ensure performance capabilities of the subsystems that are connected with the redundant subsystems [4,5]. Due consideration of the tolerances in the changes of the output parameters of the passive redundant subsystem may exert a significant influence upon the structure of its redundancy and, particularly, upon the quantity of its main components, which can be greater than unity by many times.

We will assume that quantity of the main components of the redundant subsystem is equal to the minimum quantity of the working components "m" from the total quantity "n", at which it is possible to ensure the maximum permissible deviation of its output parameter in the cases of failures of all redundant components (r = m-n) and in this case (it is obvious) its performance capability is maintained. Parameters "n" and "m" determine the structure of the passive redundancy, that is, quantity of the main components "m", quantity of the redundant components "r", and total quantity of all components "n".

For example, as concerns the fuel supply systems of the aircraft engines, it is possible to specify the maximum permissible decrease from the rated total capacity of the fuel supply pumps on the basis of the specific features of the aircraft structure, as well as on the basis of the requirement to ensure all working modes of the engine (including forced modes) during the required period of time. The same situations exist in the aircraft onboard AC/DC power supply systems, flight control actuators, power amplifiers of the aircraft control circuits etc., because of decrease of the output power, engine torque (or torsional force) over the permissible limits can cause failures of equipment. The goal of this paper is to determine the specific features, which must be taken into consideration in the course of research and development of the passive redundant subsystems of the aircraft with due consideration of the tolerances in divergence of their output parameters.

Therefore, necessity of the due consideration of tolerances is connected with the fact that the passively redundant subsystems are connected with other subsystems in a constructive way. This connection results in the following: in the cases of failures of components of the redundant subsystem, there are changes in its output parameters, however, these parameters are sent to the input of another subsystem that is connected with the redundant subsystem. Consequently, it is necessary to determine structure of the passive redundancy of the subsystem (quantities of the main components and redundant components) on the basis of necessity to ensure the performance capability of the neighboring subsystem, which is connected with the redundant subsystem.

2. KINDS OF TOLERANCES

It is worthwhile to draw a line between the specified and realizable tolerances. The specified tolerance is determined for the neighboring subsystem, which is connected with the passively redundant subsystem in a constructive way. It determines the maximum permissible deviation from the rated value (nominal value) of the input parameter of the neighboring subsystem. This deviation ensures normal operation of this subsystem. The specified tolerance is determined in accordance with the normative and technical documentation in respect of the technological item under consideration. This tolerance is specified for the continuous scale of possible tolerances of the present kind of subsystems. For example, as concerns the fuel supply systems of the aircraft engines, it determines the minimum total capacity of the working fuel supply pumps, which ensure performance capability of the aircraft engine for all working modes during the required period of time. The same situations exist in the other above-referenced subsystems that were listed in the Introduction.

The realizable tolerance is ensured by the passively redundant subsystem. It is the discrete value, which is determined by the discrete structure of the passive redundancy, that is, this tolerance is determined by the discrete values "m", "r", and "n". The realizable tolerance is characterized by the value of divergence of the output parameter from the nominal value in the case of failure (breakdown) of all redundant components (r = n – m) of the subsystem. As it follows from the abovementioned, the specified and realizable tolerances may have various values.

The specified tolerance may be determined both upwards, and downwards. The downwards tolerance (decrease of the output parameter from the nominal value) is the most significant factor for practical purposes, because of decrease of the output parameter is the most critical factor in order to ensure all working modes of the neighboring subsystems, including limiting modes of operation. Therefore, the downwards tolerance will be analyzed hereinafter. In this case, the tolerance, which is realizable in the course of the passive redundancy, must be "narrower" as compared with the specified tolerance.

3. DETERMINATION AND CHARACTERISTICS OF TOLERANCES

The realizable and the specified tolerances can be designated in the absolute and relative values. The specified tolerance in the absolute values determines maximum permissible decrease of the input parameter of the neighboring subsystem, which is connected with the passively redundant subsystem:

$$\Delta W_n = W_{nom} - W_m, \qquad (2)$$

where:

W_{nom} is nominal value of the input parameter of the neighboring subsystem;

W_m is minimum value of the input parameter of the neighboring subsystem, which ensures performance capability of this subsystem for all working modes.

The specified tolerance in the relative values is determined by the following expression:

$$dW_n = \frac{W_{nom} - W_m}{W_{nom}} \cdot 100\% \qquad (3)$$

This expression demonstrates the permissible relative value of decrease of the input parameter of the neighboring subsystem from the nominal value per each unit of its nominal value, which is expressed as a percentage. The realizable tolerance in the absolute values ΔW_p is determined (as it was mentioned above in the Introduction) by the value of divergence from the nominal value of the output parameter in the case of breakdown (failure) of all redundant components of the relevant subsystem:

$$\Delta W_p = W \cdot n - W \cdot m = r \cdot W, \qquad (4)$$

where W is the value of the output parameter of one of "n" components of the passive redundant subsystem, which work in parallel.

The realizable tolerance in the relative values is determined by the value of the ratio ΔW_p to the nominal value of the output parameter of the redundant subsystem $W \cdot n$, which is expressed as a percentage:

$$dW_p = \frac{r \cdot W}{n \cdot W} \cdot 100\% = \frac{r}{n} \cdot 100\% = \frac{n-m}{n} \cdot 100\% \qquad (5)$$

The following conditions must be complied with for the realizable and specified tolerances both in the absolute, and in the relative representations:

$$\Delta W_n \geq \Delta W_p, \qquad (6)$$

$$dW_n \geq dW_p. \qquad (7)$$

The specified and realizable tolerances in the relative representations are used more frequently. The discrete mesh of the realizable tolerances in the relative representation is of special interest for practical purposes. Therefore, hereinafter we will analyze only these tolerances. Let us draw up the Table of the realizable values of relative tolerances of various structures of the passive redundancy. This table will have the most complete view depending on the parameters "m" and "r".

Table 1: Values of the realizable relative tolerances expressed as a percentage depending on the quantity of the main components "m" and redundant components "r"

m	r										
	1	2	3	4	5	6	7	8	9	10	11
1	50.0	66.7	75.0	80.0	83.3	85.7	87.5	88.0	90.0	90.9	91.7
2	33.3	50.0	60.0	66.7	71.4	75.0	77.8	80.0	81.8	83.3	846
3	25.0	40.0	50.0	57.1	62.5	66.7	70.0	72.7	75.0	76.9	78.6
4	20.0	33.3	42.8	50.0	55.5	60.0	63.6	66.7	69.2	71.4	73.3
5	16.7	28.6	37.5	44.4	50.0	54.5	58.3	61.5	64.3	66.7	68.7
6	14.3	25.0	33.3	40.0	45.4	50.0	53.8	57.1	60.0	62.5	64.7
7	12.5	22.2	30.0	36.4	41.7	46.1	50.0	53.5	56.2	58.8	61.1
8	11.1	20.0	27.3	33.3	38.5	42.8	46.7	50.0	52.9	55.5	57.9
9	10.0	18.2	25.0	30.8	35.7	40.0	43.7	47.1	50.0	52.6	55.0

Analysis of the above-presented table makes it possible to draw the following conclusions:
1) The realizable tolerances create the sufficiently great manifoldness of possible discrete values (both greater than 50% and less than 50%);

2) Values of tolerances are closely connected with the complexity of the redundancy structures: complexity of the redundancy structures increases along with the growth of "m" and "r" values;

3) Changes of tolerances along with the growth of "m" and "r" values occur in opposite directions: along with the growth of "r" value and at the

fixed "m" value, tolerances are increased, while along with the growth of "m" value and at the fixed "r" value, tolerances are decreased;

4) In accordance with changes of "r" and "m" values, tolerances are repeated with a certain periodicity. For example, tolerance 50% is realized at r = 1 and m = 1, r = 2 and m = 2 and so on, tolerance 33.3% is realized at r = 1 and m = 2, r = 2 and m = 4 and so on.

The scale of decreasing tolerances is the most important scale because of these tolerances are the most stringent ones. Therefore, we will analyze changes of these tolerances in accordance with the principle: column by column. In this situation (as it will be clear from the following discussion), tolerances of the first column of the Table 1 (at r = 1; they create the following sequence of values: 50%; 33.3%; 25%; 20%; 16.7%; 14.3%; 12.5%; 11.1%; 10%) will be the most significant ones. We will call them as the tolerances of the first level. Therefore, the tolerances, which are determined by the second, third, and next columns, we will call as the tolerances of the second, third, and next of levels. Along with the growth of the tolerance level, redundancy structure is complicated drastically, due to the fact that this structure is determined by the quantities of the main components "m" and redundant components "r". For this very reason, tolerances of the first and second levels are the most significant ones. Joint utilization of tolerances of the first and second levels ensures creation of the more narrow mesh of the realizable tolerances. The mesh, which is created by the non-recurrent relative realizable tolerances, has the view as follows: 66.7%; 50%; 40%; 33.3%; 28.6%; 25%; 22.2%; 20%; 18.2%; 16.7%; 14.3%; 12.5%; 11.1%; 10%. Such mesh ensures close approximation to the specified tolerances.

4. N-TUPLE AND ALIQUANT PASSIVE REDUNDANCY AND THEIR INTERCONNECTION WITH TOLERANCE

The tolerances, which are presented in the Table 1, correspond to the redundancy structures with minimum multiplenesses. For example, tolerance 50% at r = 1 and m = 1 corresponds to the multipleness $K = \frac{2}{1} = 2$, tolerance 33.3% at r = 1 and m = 2 corresponds to the multipleness $K = \frac{3}{2}$ and so on. No lesser values of multiplenesses at the above-referenced tolerances exist.

The first line of the Table 1 corresponds to the subsystems with the n-tuple redundancy. At r = 1, degree of redundancy is equal to 2, at r = 2 degree of redundancy is equal to 3, while at r = 11 degree of redundancy is equal to 12. Therefore, the first line of the Table 1 implements the changes in the multipleness beginning from the simple doubling and up to the 12-tuple redundancy. The tolerances that are situated below the first line correspond to the structures with aliquant redundancy. They determine the main volume of the realizable tolerances and they are of greatest interest for researches.

Conditions for implementation of the aliquant redundancy are as follows:

1) Quantity of the main components must be more than unity (m>1);

2) Total quantity of all components must be more than quantity of the main components (n>m);

3) Any component of the redundancy structure can replace any failed component.

It is worthwhile to note that degree of redundancy of the subsystems with aliquant redundancy is always written in fractional form even if the degree of redundancy represents an integer-valued ratio. In addition, the subsystems with aliquant redundancy have an important property: indices of the failure-free operation (reliability factors) of the subsystems with aliquant redundancy are always higher than the same indices/factors of the subsystems with the n-tuple redundancy at the same values of "m" and "n". However (as it will be shown below), one must pay for this useful property: the subsystems with aliquant redundancy have critical values of probabilities of the components of the redundant subsystems that are similar to the subsystems with active redundancy, and these critical values substantially decrease the range, within which this kind of redundancy is useful [6, 7].

Each tolerance can be implemented at various values of multiplenesses of both n-tuple redundancy, and the aliquant redundancy. In this situation, its own scale (ruler) of various values of multiplenesses exists for each tolerance, and this scale/ruler begins from the minimum value. Both minimum values, and other values of the multipleness of various tolerances differ from each other substantially. Therefore, there exists the great manifoldness of various multiplenesses for various values of tolerances.

In order to ensure comparison of the failure-free operation indices at various tolerances depending on the series of increasing multiplenesses, it is reasonable to carry out unification of these various increasing multiplenesses (one way or the other). We can select the following unification parameter: value of the generalized multipleness K_i, where $i$is the sequence number of the relevant multipleness within the sequence of their increasing ratios at any values of the realizable tolerances. For example, at the tolerance of 50% the increasing row of multiplenesses has the following view: $K_1 = \frac{2}{1} = 2, K_2 = \frac{4}{2}, K_3 = \frac{6}{3}$... At the tolerance of 33.3%, the increasing row of multiplenesses has the following view: $K_1 = \frac{3}{2}, K_2 = \frac{6}{4}, K_3 = \frac{9}{6}$... At the tolerance of 25%, the increasing row of multiplenesses row of multiplenesses has the following view: $K_1 = \frac{4}{3}, K_2 = \frac{8}{6}, K_3 = \frac{12}{9}$....

For the tolerances of the first level (the main level), index i coincides with the quantity of the redundant components within the structure. Actually, quantity of the redundant components "r" within the above-described examples of three tolerances (50%, 33.3%, 25%) for K_1 are equal to, respectively, 2-1=3-2=4-3=1; for K_2 are equal to 4-2=6-4=8-6=2; for K_3 are equal to 6-3=9-6=12-9=3; and so on. Therefore, for the tolerances of the first level, index i determines quantity of the redundant components within the redundancy structure, which corresponds to this multipleness. There is no such coincidence for the tolerances of the rest levels.

Hereinafter we will use not only individual values of multiplenesses, but generalized values K_i as well. In the Table 2, we have presented the series of ten increasing multiplenesses in the individual and generalized representation, which were created for tolerances of the first level from the Table 1. In the Table 3 we have presented the similar series of multiplenesses for the following tolerances of the second level, which are non-recurrent with the tolerances of the first level: 66.7; 40%; 28.6%; 22.2%, 18.2%.

Table 2: Values of multiplenesses, which correspond to the values of the realizable relative tolerances of the first level that were presented in the Table 1.

dW %	Redundancy multiplenesses K_i									
	K_1 (r = 1)	K_2 (r = 2)	K_3 (r=3)	K_4 (r=4)	K_5 (r=5)	K_6 (r=6)	K_7 (r=7)	K_8 (r=8)	K_9 (r=9)	K_{10} (r = 10)
50	2	$\frac{4}{2}$	$\frac{6}{3}$	$\frac{8}{4}$	$\frac{10}{5}$	$\frac{12}{6}$	$\frac{14}{7}$	$\frac{16}{8}$	$\frac{18}{9}$	$\frac{20}{10}$
33.3	$\frac{3}{2}$	$\frac{6}{4}$	$\frac{9}{6}$	$\frac{12}{8}$	$\frac{15}{10}$	$\frac{18}{12}$	$\frac{21}{14}$	$\frac{24}{16}$	$\frac{27}{18}$	$\frac{30}{20}$
25	$\frac{4}{3}$	$\frac{8}{6}$	$\frac{12}{9}$	$\frac{16}{12}$	$\frac{20}{15}$	$\frac{24}{18}$	$\frac{28}{21}$	$\frac{32}{24}$	$\frac{36}{27}$	$\frac{40}{30}$
20	$\frac{5}{4}$	$\frac{10}{8}$	$\frac{15}{12}$	$\frac{20}{16}$	$\frac{25}{20}$	$\frac{30}{24}$	$\frac{35}{28}$	$\frac{40}{32}$	$\frac{45}{36}$	$\frac{50}{40}$
16.7	$\frac{6}{5}$	$\frac{12}{10}$	$\frac{18}{15}$	$\frac{24}{20}$	$\frac{30}{25}$	$\frac{36}{30}$	$\frac{42}{35}$	$\frac{48}{40}$	$\frac{54}{45}$	$\frac{60}{50}$
14.3	$\frac{7}{6}$	$\frac{14}{12}$	$\frac{21}{18}$	$\frac{28}{24}$	$\frac{35}{30}$	$\frac{42}{36}$	$\frac{49}{42}$	$\frac{56}{48}$	$\frac{63}{54}$	$\frac{70}{60}$
12.5	$\frac{8}{7}$	$\frac{16}{14}$	$\frac{24}{21}$	$\frac{32}{28}$	$\frac{40}{35}$	$\frac{48}{42}$	$\frac{56}{49}$	$\frac{64}{56}$	$\frac{72}{63}$	$\frac{80}{70}$
11.1	$\frac{9}{8}$	$\frac{18}{16}$	$\frac{27}{24}$	$\frac{36}{32}$	$\frac{45}{40}$	$\frac{54}{48}$	$\frac{63}{56}$	$\frac{72}{64}$	$\frac{81}{72}$	$\frac{90}{80}$

10	$\frac{10}{9}$	$\frac{20}{18}$	$\frac{30}{27}$	$\frac{40}{36}$	$\frac{50}{45}$	$\frac{60}{54}$	$\frac{70}{63}$	$\frac{80}{72}$	$\frac{90}{81}$	$\frac{100}{90}$

The first cell of the Table 2 at the tolerance of 50%, as well as the first cell of the Table 3 at the tolerance of 66.7% correspond to the n-tuple redundancy, while the rest cells of two above-referenced Tables correspond to the aliquant redundancy.

Table 3: Values of multiplenesses, which correspond to the values of the realizable relative tolerances of the second level, which were presented in the Table 1.

dW %	Redundancy multiplenesses K_i									
	K_2	K_3	K_4	K_5	K_6	K_7	K_8	K_9	K_{10}	K_{11}
66.7	3	$\frac{6}{2}$	$\frac{9}{3}$	$\frac{12}{4}$	$\frac{15}{5}$	$\frac{18}{6}$	$\frac{21}{7}$	$\frac{24}{8}$	$\frac{27}{9}$	$\frac{30}{10}$
40	$\frac{5}{3}$	$\frac{10}{6}$	$\frac{15}{9}$	$\frac{20}{12}$	$\frac{25}{15}$	$\frac{30}{18}$	$\frac{35}{21}$	$\frac{40}{24}$	$\frac{45}{27}$	$\frac{50}{30}$
28.6	$\frac{7}{5}$	$\frac{14}{10}$	$\frac{21}{15}$	$\frac{28}{20}$	$\frac{35}{25}$	$\frac{42}{30}$	$\frac{49}{35}$	$\frac{56}{40}$	$\frac{63}{45}$	$\frac{70}{50}$
22.2	$\frac{9}{7}$	$\frac{18}{14}$	$\frac{27}{21}$	$\frac{36}{28}$	$\frac{45}{35}$	$\frac{54}{42}$	$\frac{63}{49}$	$\frac{72}{56}$	$\frac{81}{63}$	$\frac{90}{70}$
18.2	$\frac{11}{9}$	$\frac{22}{18}$	$\frac{33}{27}$	$\frac{44}{36}$	$\frac{55}{45}$	$\frac{66}{54}$	$\frac{77}{63}$	$\frac{88}{72}$	$\frac{99}{81}$	$\frac{110}{90}$

One of the most important specific features of the passive redundancy structures of the aircraft subsystems with due consideration of the tolerances is the necessity to ensure decrease of the weight and dimensional characteristics (as well as decrease of the monetary characteristics) of their components along with further aggravation of tolerances, as well as along with further increase of multiplenesses as compared with the similar characteristics of the non-redundant subsystem. Actually, replacement of the non-redundant subsystem, which consists of the single component with nominal value of the output parameter W_{nom}, by the passive redundant subsystem, which consists of "n" similar components, which work in parallel, causes proper decrease of their output parameters W by "n" times:

$$W = \frac{W_{nom}}{n} \qquad (8)$$

With reference to the above-stated, it is possible to assume that replacement of the non-redundant subsystem by the redundant subsystem, which consists of "n" components, in order to increase its failure-free operation would not result in the substantial increase of the cost, as well as increase of the weight and dimensional characteristics of the redundant subsystem as a whole.

The important questions arise: "How parameters of the failure-free operation of the passive redundant subsystem are changed depending on the multipleness?" "Which value of multipleness (to the maximum extent) complies with the requirements in respect of the failure-free operation at the given value of the specified tolerance?" Answers to these questions depend on the selected criterion, as well as on the technical requirements to the subsystem under consideration (requirements in respect of the failure-free operation to the subsystem as a whole and to its components, to the specified tolerance and other requirements), and these answers will be analyzed in our next paper.

5. ANALYSIS OF INFLUENCE OF VALUES OF TOLERANCES AND MULTIPLENESSES UPON THE VALUES OF THE CRITICAL PROBABILITIES OF COMPONENTS OF THE PASSIVE REDUNDANT SUBSYSTEMS OF THE AIRCRAFT

Critical values of probabilities of components are connected with the indices of the failure-free operation of the non-redundant and of the passive redundant subsystems of the aircraft. We will use probabilities of their failure-free operation (they are often referred to as the functions of the failure-free operation) as the indices of the failure-free operation of these subsystems.

Let us assume that $P_c(t_з) = P_c$ is the probability of failure-free operation of the passive redundant subsystem during performance of the task, while $p(t_з) = p$ is the probability of failure-free operation of the non-redundant subsystem (which is the component of the passive redundant subsystem) during performance of the task. It is obvious that function of the failure-free operation of the passive redundant subsystem P_c depends on the redundancy structure (that is, on the parameters "m" and "n"), as well as on the failure-free operation of its components p:

$$P_c = P_c(p, n, m). \qquad (9)$$

There exist many various methods (probabilistic methods, logical-and-probabilistic methods, and logical methods) for calculation of the function (9). Let us select the probabilistic method, which is based on the binomial distribution law. In accordance with this law, probability of the failure-free operation of "m" components from the total quantity of "n" components of the passive redundant subsystem is determined with the help of the following formula:

$$P_{m,n} = C_n^m \cdot p^m \cdot (1-p)^{n-m}. \qquad (10)$$

The function of the failure-free operation (9) is determined as the probability of failure-free operation of not less than "m" components from the total quantity of "n" components:

$$P_c = \sum_{i=m}^{n} C_n^i \cdot p^i \cdot (1-p)^{n-i}. \qquad (11)$$

As concerns the subsystems with the structures of the n-tuple redundancy of any multipleness (the first line of the Table 1), function of the failure-free operation P_c is always higher than the function of the failure-free operation of the non-redundant subsystem within the open interval: 0<p<1. It follows from the statement that the aircraft subsystem with the n-tuple passive redundancy is in working order, if at least one its component is in working order [3]. In this case, quantity of the main components "m" = 1, while the function of the failure-free operation of the subsystems with the n-tuple passive redundancy is determined by the following expressions:

$$P_c = \sum_{i=1}^{n} C_n^i \cdot p^i \cdot (1-p)^{n-i}, \qquad (12)$$

$$P_c = 1 - q^n, \text{ where } q = 1 - p \qquad (13)$$

It follows from the expression (13) that the following inequality is valid for the subsystems with the n-tuple passive redundancy:

$$P_c = 1 - q^n > p = 1 - q. \qquad (14)$$

As concerns the subsystems with the structures of the aliquant passive redundancy (the second line, as well as the underlying lines of tolerances of the Table 1), quantity of the main components (as it was already pointed out) is always higher than the unity: m>1. Therefore, summation in the formulas of the failure-free operation (11) for any tolerances and multiplenesses is performed beginning from the values of the index i, which is equal to two, three, and four etc. For example, for three tolerances of the first level at the minimum multiplenesses (dW=33.3%; dW=25%; dW=20%) three formulas of the failure-free operation are as follows, respectively:

$$P_c^1 = \sum_{i=2}^{3} C_3^i \cdot p^i \cdot (1-p)^{3-i}, K_1 = \frac{3}{2}; \qquad (15)$$

$$P_c^2 = \sum_{i=3}^{4} C_4^i \cdot p^i \cdot (1-p)^{4-i}, K_1 = \frac{4}{3}; \qquad (16)$$

$$P_c^3 = \sum_{i=4}^{5} C_5^i \cdot p^i \cdot (1-p)^{5-i}, K_1 = \frac{3}{2}; \qquad (17)$$

As it was stated in [1], the following relationship is valid for any structures of the aliquant redundancy:

$P_c(p,n,m) < p$, at p values, which are close to zero, and

$P_c(p,n,m) > p$, at p values, which are close to unity.

In this situation, dependences $P_c(p,n,m)$ and $p(p)$ have one point of intersection within the open interval (0 – 1), while equation:

$$P_c(p,n,m) = p \quad (18)$$

has one real root. Value of the probability, which corresponds to this root, is referred to as the critical value and it is denoted as p_{kr}.

Therefore, in the case of the aliquant passive redundancy, there exist critical values of probabilities of components of the redundant subsystems, and these critical values brake up the interval (0 – 1) and divide it by two subintervals:

– precritical subinterval (0 -p_{kr}), where $P_c(p,n,m) < p$ and passive redundancy is not useful;

– postcritical subinterval (p_{kr}- 1), where $P_c(p,n,m) > p$ and passive redundancy is useful.

Formulas (15), (16), and (17) demonstrate that they have the lesser quantities of the summands (by one summand, two summands, and three summands, respectively) as compared with the formula (12) for the n-tuple passive redundancy. This fact points to the increase of the values of critical probabilities along with further decrease (tightening) of tolerances. It is possible to use analytical and numerical methods of calculation of critical values p_{kr}. The analytical methods are too laborious ones and they may be of academic interest only (to a greater extent). These methods are limited to the reduction of order of the algebraic equations (not higher than third or fourth degree), for which there exist acceptable formulas for calculation of roots. Therefore, the numerical methods are more important ones for practical purposes.

In the Table 4, we have presented the results of calculations of critical values of probabilities of the components of the passive redundant subsystems p_{kr} for the tolerances of the first level and for the increasing multiplenesses within each of these tolerances, while in the Table 5 we have presented similar results for the tolerances of the second level.

Table 4: Critical values of probabilities p_{kr} for 11 increasing multiplenesses, which correspond to 9 realizable tolerances of the first level

dW	Redundancy multiplenesses K_i										
	K_1 (r = 1)	K_2 (r = 2)	K_3 (r=3)	K_4 (r=4)	K_5 (r=5)	K_6 (r=6)	K_7 (r=7)	K_8 (r=8)	K_9 (r=9)	K_{10} (r = 10)	K_{11} (r = 11)
50	-	233	348	396	422	438	449	456	462	467	470
33.3	500	653	679	687	690	691	691	691	691	691	691
25	768	803	804	801	799	796	794	792	790	789	787
20	869	871	865	859	855	851	848	846	844	842	840
16.7	917	908	900	893	889	885	882	879	877	875	873
14.3	943	930	921	915	910	907	904	901	899	897	896
12.5	958	945	936	930	926	922	919	917	915	913	912
11.1	968	955	947	941	937	934	931	929	927	925	924
10	975	963	955	950	946	943	940	938	936	934	933

The first cells of the first lines in the Tables 4 and 5 correspond to the n-tuple redundancy and hereinafter they will not be analyzed due to the absence of critical values p_{kr} (for these cells) within the interval of probabilities (0 – 1). For the sake of space, in each cell of the Table 4 we have presented only three decimals in the calculated values of critical probabilities.

Table 5: Critical values of probabilities p_{kr} for 9 increasing multiplenesses, which correspond to 5 realizable tolerances of the first level

dW	Redundancy multiplenesses K_i								
	K_2	K_3	K_4	K_5	K_6	K_7	K_8	K_9	K_{10}
66.7	-	0.084	0.158	0.197	0.221	0.237	0.248	0.257	0.263
40	0.500	0.579	0.596	0.602	0.605	0.607	0.608	0.608	0.609
28.6	0.745	0.755	0.753	0.750	0.748	0.746	0.745	0.743	0.742
22.2	0.843	0.835	0.828	0.823	0.819	0.816	0.814	0.812	0.810
18.2	0.892	0.878	0.870	0.864	0.860	0.857	0.855	0.853	0.851

By way of illustration of the results, which were presented in the Tables 4 and 5, Figures 1 and 2 present the dependences of critical probabilities p_{kr} on the multiplenesses for two ranges of tolerances of the first level. Figure 3 presents dependences p_{kr} on the multiplenesses for the tolerances of the second level.

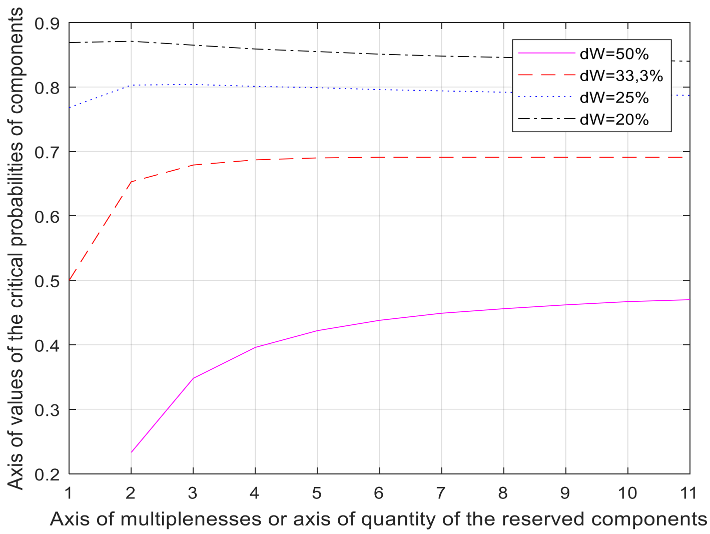

Figure 1: Dependences p_{kr} on K_i for big tolerances of the first level

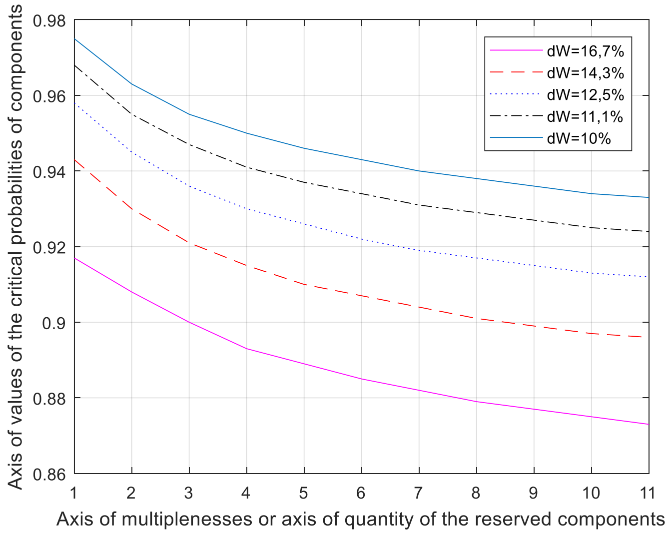

Figure 2: Dependences p_{kr} on K_i for small tolerances of the first level

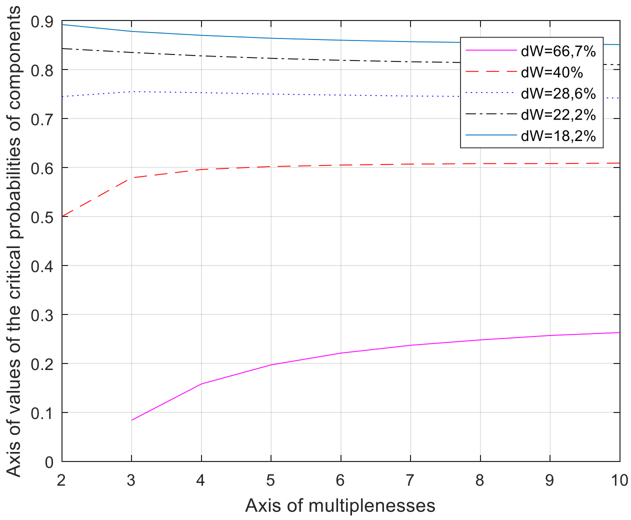

Figure 3: Dependences p_{kr} on K_i for values of tolerances of the second level

Analysis of the above-presented dependences for the realizable tolerances of the first and second levels makes it possible to draw the following conclusions:

1) The range of probabilities of the components, within which it is useful to implement this kind of redundancy, depends on three factors: level of tolerance, value of tolerance, and multipleness of redundancy;

2) At any level of tolerance and at any multipleness of redundancy along with futher decrease (tightening) of values of tolerances, critical values of probabilities p_{kr} increase monotonically;

3) For each level of tolerances, there exist equilibrium values dW_p, at which critical values p_{kr} are practically unchanged depending on the multiplenesses K_r. For example, for the tolerances of the first level: dW_p=25%, while for the tolerances of the second level: dW_p=28.6%;
4) Pattern of changes of critical probabilities (depending on the multipleness values) changes drastically: at the tolerances, which do not exceed dW_p, critical values increase, while at the tolerances, which exceed dW_p, critical values decrease.

6. CONCLUSIONS

The present paper is the first part of the research, which is devoted to the analysis and synthesis of the passive redundant subsystems of the aircraft with due consideration of the tolerances. The goal of this paper is to determine the specific features of the above-referenced subsystems, that is, the features, which must be taken into consideration in the course of analysis and synthesis of such subsystems. The following results were obtained:

1) It was shown that it is necessary to take into consideration two kinds of tolerances (specified tolerances and realizable tolerances), which are essentially different in respect of the method of their representation and assignment. The first tolerances are specified for the continuous scale of values, while the second ones are implemented for the discrete scale, which is determined by the redundancy structure;

2) It was established that it is possible to assign these tolerances both for decrease, and for increase of the output parameter from the nominal value, however, in the course of this research we have only selected the tolerances of decrease from the nominal value, because of they are more significant ones for practical purposes;

3) It was established that passive redundancy with due consideration of the tolerances is implemented with the help of two essentially different methods: n-tuple and aliquant passive redundancy;

4) It was demonstrated that the tolerances, which are implemented with the help of the methods of the aliquant redundancy, are the most significant ones for practical purposes in respect of their quantity, diversity of values, and indices of the failure-free operation;

5) It was demonstrated that subsystems with the aliquant passive redundancy have values of probabilities of the components, which are referred to as the critical values p_{kr}, which determine existence of the areas (0 – p_{kr}), where redundancy is not useful;

6) It was established that along with futher decrease of tolerances (at minimum multiplenesses), values of critical probabilities increase rapidly, thus decreasing the areas (p_{kr} – 1), where redundancy is useful;

7) It was established that pattern of changes of critical values p_{kr}depending on the multipleness values changes drastically. For example, at the tolerances, which do not exceed dW_p, critical values increase, while at the tolerances, which exceed dW_p, critical values decrease. Similar pattern of changes of p_{kr} exists for the tolerance 28.6 for the tolerances of the second level.

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