

# ANALYSIS OF THE FAILURE-FREE OPERATION OF THE PASSIVE REDUNDANT SUBSYSTEMS OF THE AIRCRAFT WITH DUE CONSIDERATION OF THE TOLERANCES

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## ABSTRACT

The goal of this paper is to examine the failure-free operation of the passive redundant subsystems of the aircraft with due consideration of the tolerances for decrease of their output parameters in the cases of failures of the components. The following factors were investigated: influence of values of the failure-free operation of the components, as well as influence of tolerances and multiplicity of redundancy upon the failure-free operation of the passive redundant subsystems as a whole. It was demonstrated that in the cases where relative tolerances do not exceed 25%, increase of the multiplicity of redundancy ensures essential growth of the failure-free operation of the subsystems within the postcritical areas of probabilities of the components.

## KEYWORDS

Failure-free operation of subsystems, failure-free operation of components, degree of redundancy, aliquant redundancy, critical probability of components, tolerance of decrease of the output parameters of subsystems.

## 1. INTRODUCTION

One of the important problems in the course of research and development of the prospective equipment samples is to ensure compliance with the requirements in respect of reliability of the nonrepairable subsystems of the aircraft in the cases of sudden failures. This problem is especially important for such subsystems, as power amplifiers and flight control actuators of the aircraft control circuits, fuel supply systems of the aircraft engines, the aircraft onboard AC/DC power supply systems, because of failure-free operation of the entire aircraft is in the essential dependence on these subsystems. Therefore, this paper analyses those aircraft subsystems, which do not permit work stoppages and for which very high requirements are established in respect of their failure-free operation. One of the main means of increase of the failure-free operation of such subsystem is their passive (standby) redundancy, which ensures continuous operation of these subsystems [1].

The important specific feature of the above-listed subsystems is availability of engineering constraints in respect of divergence of their output parameters in the cases of failures of various components. These constraints are established in order to ensure normal operation of the subsystem, which is connected with the passively redundant subsystem and to which such output parameters are supplied [2, 3].

List of such output parameters is as follows: total productivity of the fuel supply pumps of the aircraft engines, total output power of synchronous generators of the aircraft onboard AC power supply system, and so on. The fuel supply system is connected with the engine; therefore, in the cases of failures of the permissible quantity of fuel supply pumps it must ensure all working modes of the engine. The same situation exist in the following connected subsystems: power amplifier of the aircraft control circuit – flight control actuator, flight control actuator – control device of the aircraft, and so on

The goal of this paper is to determine the model for estimation of the failure-free operation of the passive redundant subsystems with due consideration of the tolerances, as well as to investigate changes of the failure-free operation indices depending on the values of those parameters, which are included to this model. This model must be one of the tools of the design engineers in the course of designing the highly

reliable subsystems that belong to the class under consideration.

Results of this paper are as follows: index of the failure-free operation of the passive redundant subsystems was selected; factors, which have substantial influence upon this index and which determine model of investigation were revealed; formal characterization of these factors was made; the problem for analysis of the passive redundant subsystems of the aircraft was formulated with due consideration of the relevant tolerances; influence of substantial parameters of this model upon the index of the failure-free operation was investigated; conclusions in respect of the obtained results were made. The investigations, which were performed in the course of this work, are investigations of the methodological nature and they are not connected with any specific subsystem of any aircraft.

## 2. FORMAL CHARACTERIZATION OF THE MODEL OF INVESTIGATION

It is necessary to ensure redundancy of the certain subsystems that belong to the class under consideration in the case, where these subsystems can not comply with the requirements in respect of the failure-free operation without such redundancy. In the case of the passive redundancy, the initial subsystem is replaced by several similar subsystems on the condition that sum of output parameters of these subsystems (that are components of the subsystem, redundancy of which is ensured) would be equal to the output parameter of the initial subsystem. It follows that in the course of the passive redundancy the weight and dimensional characteristics and cost of certain components of the passive redundant subsystems must be substantially lower than the same characteristics of a non-redundant subsystem. However, the entire redundant subsystem may have a greater cost and the bigger weight and dimensional characteristics as compared with a non-redundant subsystem. For the most part, these characteristics are determined by the rationality of structure of the relevant components, as well as by the fabricability of production of these components.

In accordance with the procedure of the systematic approach, let us to create the model of investigation through selection of the factors that have substantial influence upon the index of the failure-free operation of a subsystem provided that only sudden independent failures can occur

within the class of subsystems under consideration. Let us determine that the best index (criterion) of the failure-free operation is a probability of the failure-free operation of a subsystem during performance of the task (which is often referred to as the function of reliability (function of the failure-free operation) due to importance of this index. Analysis of the specific features of the passive redundant subsystems of the aircraft demonstrates that there are the following factors that have the most essential influence upon the index of the failure-free operation: redundancy structure, failure-free operation of components of the redundant subsystem, values of critical probabilities of these components, as well as specified tolerance and realizable tolerance.

Let us perform a formal characterization of the accepted criterion and other essential factors. Introduce the following designations and assume that  $T$  is a random time of normal operation of a subsystem before the failure, while  $t_3$  is the time of performance of a task by this subsystem. Then probability of the failure-free operation of the redundant subsystem during performance of this task is determined by the following expression:

$$P_c(t_3) = P(T > t_3) = P_c \quad (1)$$

while probability of the failure-free operation of a component of the redundant subsystem is determined by the following expression:

$$p(t_3) = P(T > t_3) = p \quad (2)$$

Structure of redundancy is determined by the quantity of the main components "m", quantity of the redundant components "r", and total quantity of components "n", which are connected with each other by the following equality:

$$m + r = n \quad (3)$$

It follows from (3) that in order to determine the redundancy structure, it is sufficient to specify any two of three parameters. Let us determine "m" and "n" parameters, because of they determine not only the redundancy structure, but the degree of redundancy as well:

$$K = \frac{n}{m} \quad (4)$$

Parameter "m" determines minimum permissible quantity of the working components of the passive redundant subsystem on the basis of the required tolerance. It is worthwhile to draw a line between the specified and realizable tolerances. They can be designated in the absolute and relative representation. Tolerances in the relative representation are more preferable. The specified tolerance in the relative representation is determined by the following expression:

$$dW_n = \frac{W_{nom} - W_m}{W_{nom}} \cdot 100\% \quad (5)$$

where:

$W_{nom}$  is the nominal value of the output parameter of the passive redundant subsystem;

$W_n$  is the minimum permissible value of the output parameter of the passive redundant subsystem.

The realizable tolerance in the relative representation is determined by the following expression:

$$dW_p = \frac{n-m}{n} \cdot 100\% \quad (6)$$

It is necessary to select parameter  $dW_p$  from the mesh of the realizable tolerances in such a manner in order to ensure compliance with the following condition:

$$dW_n \geq dW_p \quad (7)$$

Each realizable tolerance can be determined with the help of various values of multiplenesses of redundancy, which create the scale of increasing multiplenesses, beginning from the minimum value of multiplenesses. Let us draw the line between various realizable tolerances in respect of their levels. The level of tolerance is determined by the quantities of the redundant components "r" at the minimum multipleness, while the set (aggregate) of the realizable tolerances is formed in the course of variation of quantity of the main components "m". Then, tolerances of the first level at  $m = 1, 2, 3, \dots$  are equal to:

$$dW_p = \frac{1}{1+1} \cdot 100\% = 50\%, \quad dW_p = \frac{1}{1+2} \cdot 100\% = 33,3\%$$

$dW_p = \frac{1}{1+3} \cdot 100\% = 25\%$ , and so on. The realizable tolerances of various levels can coincide with each other.

The realizable tolerances of the second level, which do not coincide with the tolerances of the first level, have the following values:  $dW_p = 40\%$

;  $dW_p = 28,6\%$ ;  $dW_p = 22,2\%$ ;  $dW_p = 18,2\%$ , and so on.

Therefore, hereinafter we will limit our investigations in respect of only a limited quantity of the biggest tolerances of the first and second levels.

In order to ensure comparison of values of the criterion  $P_c$  in respect of various values of the multiplenesses, which are substantially different at various tolerances, it is reasonable to use generalized designations of these tolerances  $K_i$ . Index  $i$  specifies the sequential number of the multipleness within the sequence of their increasing ratios at any values of the realizable tolerances. Actual values of multiplenesses  $K_i$  are calculated in accordance with the parameters "r" and "n" of the realizable tolerance  $dW_p = \frac{r}{n} \cdot 100\%$  and the value of index  $i$  of the sequential

number of the multipleness in accordance with the formula  $K_i = \frac{n-i}{(n-r)^i}$ :

– for the tolerances of the first level  $i=1, 2, 3 \dots$

– for the tolerances of the second level  $i=2, 3, 4 \dots$

The critical probability  $p_{kr}$  is realized in the case of the aliquant passive redundancy of subsystems with due consideration of the tolerances. Value  $p_{kr}$  determines the postcritical area ( $p_{kr} - 1$ ), within which the failure-free operation of the redundant subsystem is higher as compared with the failure-free operation of the non-redundant subsystem [4,5].

### 3. STATEMENT OF PROBLEM

In the course of development of the passive redundant subsystems with due consideration of the tolerances, a design engineer must have proper information concerning influence of essential factors upon the index of the failure-free operation. Therefore, it is necessary to investigate the dependence of the criterial function from such essential parameters. Consequently, it is also necessary to solve the computational problem, which in the formalized form is presented as follows:

$$P_c = P_c(p, m, n, dW_n, dW_p, p_{kr}) \quad (8)$$

under following constraints:

$$dW_n \geq dW_p \quad (9)$$

$$p > P_{kr} \quad (10)$$

$$m, n > 0. \quad (11)$$

and they are whole numbers. In words, this problem is formulated as follows: it is necessary to investigate the dependence (8) from the essential parameters under constraints (9) – (11). It should be noted that the realizable tolerance depends on the specified tolerance, as well as that the realizable tolerance is to be calculated with the help of inequality (9). The realizable tolerance is more important factor for practical calculations, because of this tolerance determines the discrete structure of redundancy (parameters "n" and "m"), as well as the value of the critical probabilities of components of the redundant subsystem. Therefore, we will analyse only four parameters  $p, n, m, n, dW_p$ , if we will accept the redundancy multiplicity (or multiplicity of redundancy)  $K_i = \frac{n \cdot i}{(n-r) \cdot i}$ .

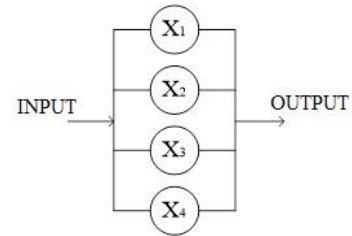
#### 4. METHOD FOR SOLVING THE PROBLEM

In order to calculate index of the failure-free operation (8) under constraints (9)–(11) it is possible to apply the universal approach, which is often used for calculation of indices of reliability of various technical systems, which have complicated structures. This approach is connected with development of the reliability structure diagram (RSD). The reliability structure diagram of subsystems is the directed graph, which has ingress point and egress point. In this case, the subsystem is considered workable, if there exists at least one workable route from input to the output of the RSD.

In accordance with such approach, calculation of the failure-free operation index (8) is connected with development of the RSD of the passive redundant subsystem, as well as with calculation of the probability in respect of operability of at least a single route from input to the output within this RSD. Therefore, solving of the above-stated problem is connected with calculation of the required index (8) with due consideration of the constraint (9)–(11) in accordance with the reliability structure diagram and known indices of the failure-free operation of relevant components.

The reliability structure diagrams of any systems and their subsystems (except for the network systems) at any methods of redundancy have a view of the parallel-series connection of their components. Therefore, they can be always separated by sections with parallel and series connection of components. This fact simplifies development of the reliability structure diagrams. In the course of development of these sections, two principles are used. The RSD section represents series connection of components, if workable operation of all components of this section is required for normal operation of this section. The RSD section represents parallel connection of components, if workable operation of at least one of its components is required for normal operation of this section.

In the course of solving the problem, it is necessary to implement the subsystems with n-tuple passive redundancy and aliquant passive redundancy, while RSDs of these subsystems are substantially different as compared with each other. The RSD of the subsystems with the n-tuple redundancy and with due consideration of the tolerances are implemented at "m" = 1 and at any "n" > "m". It follows that the RSD of the subsystems with the n-tuple redundancy represents parallel connection of all its components. For example, fuel supply system of the engine the aircraft, which includes 4 fuel pumps and ensures normal operation of the engine for all working modes at the workable operation of at least one fuel pump, has the view of the fourfold redundancy. It is shown in Figure 1.

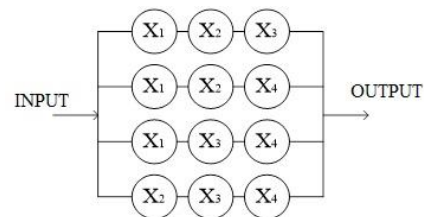


**Figure 1:** The reliability structure diagram of the fuel supply system of the aircraft engines, at  $m = 1$  and  $n = 4$ .

Within the above Figure:  $X_i$  is the event of operability of the  $i$ -th pump. In accordance with the already accepted assumption concerning independency of failures, the RSD, which is shown in Figure 1, is the sum of the joint and independent events  $X_i$ ,  $i = 1, 2, 3, 4$ . Assuming that  $P(X_i) = p$ , it is very simple to calculate index of the failure-free operation (8) with the help of the RSD through additional probability:

$$P_c = 1 - p^4 \quad (12)$$

It is possible to implement the RSDs of the subsystems with the aliquant redundancy at  $m \geq 2$  and  $n > m$ . It follows that operability of at least one route from all possible routes is required for normal operation of subsystems, while each route represents one of combinations of  $n$  components  $X_i$  within  $m$  components. For example, fuel supply system of the aircraft engine, which includes four fuel pumps and ensures normal operation of the engine for all working modes in the case of operability of any three of pumps, is shown in Figure 2.



**Figure 2:** The reliability structure diagram of the fuel supply system of the aircraft engines, at  $m = 3$  and  $n = 4$ .

The RSD, which is shown in Figure 2, determines the unknown index  $P_c$  as the probability of the sum of the joint and dependent complicated events  $A_1, A_2, A_3, A_4$ , which determine the operability of relevant routes. Formula of calculation of the failure-free operation is as follows:

$$P_c = P\left(\sum_{i=1}^4 A_i\right) \quad (13)$$

where:

$$A_1 = X_1 \cdot X_2 \cdot X_3, A_2 = X_1 \cdot X_2 \cdot X_4, A_3 = X_1 \cdot X_3 \cdot X_4, A_4 = X_2 \cdot X_3 \cdot X_4,$$

While

$$P(X_i) = p, \quad i=1, 2, 3, 4.$$

Various methods of solving the expression (13) result in different calculation formulas [6]. The simplest formula is as follows (14):

$$P_c = \sum_{i=3}^4 C_4^i p^i \cdot (1-p)^{4-i} \quad (14)$$

It makes it possible to sum up the terms of expansion by the binomial theorem  $(p + q)^4$ , which are included to various groups in respect of the same powers  $p$ . These terms include not less than three terms  $p$  (as multiplicands) in accordance with the conditions of the example and according to the RSD.

In the general case, formula for calculation of the failure-free operation index (8) for the subsystems with aliquant redundancy is as follows:

$$P_c = \sum_{i=m}^n C_n^i p^i \cdot (1-p)^{n-i} \tag{15}$$

**5. INVESTIGATION OF DEPENDENCE OF THE FAILURE-FREE OPERATION INDEX FROM THE MULTIPLINESSES OF REDUNDANCY FOR VARIOUS PROBABILITIES OF COMPONENTS, AS WELL AS FOR VARIOUS REALIZABLE TOLERANCES**

Let us investigate the dependence  $P_c$  from the multiplenesses of redundancy for various values of the failure-free operation of components, as well as for various values of the realizable tolerances of the passive redundant subsystems. Due to the necessity to perform researches within the range of probabilities  $p > p_{kr}$  (while  $p_{kr}$  depends from the multiplenesses of redundancy, as well as from the values of the realizable

tolerances), it is necessary (from the very beginning) to determine maximum value of the critical probability for each tolerance

$$p_{krm} = \max_i p_{kr}(K_i) \tag{16}$$

Value  $p_{krm}$  determines initial values of the scaleable probabilities within the range ( $p_{krm}$  - RB). Here RB denotes the right hand border of the range, and this RB may be changed depending on the relevant tolerance. It is approximately determined by the value of probability of a component of the redundant subsystem on the condition that at least one value from the multiplicity of indices of the failure-free operation will achieve the probability, which would be close to 1 (which would exceed 0.99) depending on the multipleness. In accordance with the tolerances of the first level, which were analysed in this paper, value of the RB changes from 0.750 up to 0.970, while for the tolerances of the second level this value changes from 0.850 up to 0.930. In order to analyse behaviour of  $P_c$ , we will specify 7 approximately uniformly distributed probabilities of components of the redundant subsystem for each tolerance within this range. Results of preparation of the initial data and calculations of index  $P_c$  in accordance with these data for the tolerances of the first level we have presented in the Table 1.

**Table 1:** Values of the failure-free operation  $P_c$  of subsystems depending on  $K_i$ ,  $p$  and tolerances of the first level  $dW_p$

$dW_p$ %	$p$	Redundancy multiplenesses $K_i$									
		$K_1$	$K_2$	$K_3$	$K_4$	$K_5$	$K_6$	$K_7$	$K_8$	$K_9$	$K_{10}$
50	0.470	0.719	0.641	0.598	0.569	0.547	0.530	0.515	0.502	0.491	0.48
	0.500	0.75	0.687	0.656	0.637	0.623	0.613	0.605	0.598	0.593	0.588
	0.550	0.797	0.758	0.745	0.740	0.738	0.739	0.741	0.744	0.747	0.751
	0.600	0.840	0.821	0.621	0.826	0.834	0.842	0.850	0.858	0.865	0.872
	0.650	0.877	0.873	0.883	0.894	0.905	0.915	0.925	0.933	0.940	0.947
	0.700	0.910	0.916	0.929	0.942	0.953	0.961	0.968	0.974	0.979	0.983
	0.750	0.937	0.949	0.962	0.973	0.980	0.986	0.990	0.992	0.995	0.996
33.3	0.691	0.773	0.727	0.709	0.700	0.695	0.692	0.691	0.691	0.692	0.693
	0.725	0.815	0.789	0.785	0.787	0.791	0.797	0.803	0.810	0.816	0.823
	0.750	0.844	0.831	0.834	0.842	0.852	0.861	0.870	0.879	0.887	0.894
	0.775	0.871	0.868	0.878	0.890	0.901	0.912	0.921	0.930	0.937	0.944
	0.800	0.896	0.901	0.914	0.927	0.939	0.949	0.957	0.964	0.970	0.974
	0.825	0.919	0.929	0.944	0.956	0.966	0.973	0.979	0.984	0.987	0.990
	0.850	0.939	0.953	0.966	0.976	0.983	0.988	0.992	0.994	0.996	0.997
	0.796	0.813	0.788	0.784	0.786	0.791	0.797	0.803	0.810	0.816	0.823

25	0.815	0.842	0.829	0.833	0.841	0.850	0.860	0.869	0.878	0.886	0.893
	0.830	0.863	0.859	0.868	0.879	0.890	0.901	0.911	0.919	0.927	0.934
	0.845	0.884	0.886	0.898	0.911	0.923	0.934	0.943	0.950	0.957	0.963
	0.860	0.903	0.911	0.925	0.938	0.949	0.959	0.966	0.972	0.977	0.981
	0.875	0.921	0.933	0.947	0.959	0.969	0.976	0.982	0.986	0.989	0.992
	0.890	0.938	0.951	0.965	0.975	0.982	0.988	0.991	0.994	0.996	0.997
20	0.871	0.873	0.872	0.882	0.894	0.906	0.917	0.926	0.935	0.942	0.949
	0.880	0.888	0.891	0.904	0.917	0.929	0.939	0.948	0.956	0.962	0.968
	0.890	0.903	0.912	0.926	0.939	0.950	0.959	0.967	0.973	0.978	0.982
	0.900	0.919	0.930	0.944	0.957	0.967	0.974	0.980	0.985	0.988	0.991
	0.910	0.933	0.946	0.960	0.971	0.979	0.985	0.989	0.992	0.994	0.996
	0.920	0.946	0.960	0.973	0.982	0.988	0.992	0.994	0.996	0.997	0.998
16.7	0.930	0.958	0.972	0.982	0.989	0.993	0.996	0.998	0.998	0.999	0.999
	0.917	0.917	0.929	0.943	0.956	0.966	0.973	0.979	0.984	0.987	0.990
	0.925	0.931	0.944	0.959	0.970	0.978	0.984	0.988	0.991	0.994	0.995
	0.930	0.939	0.953	0.967	0.977	0.984	0.989	0.992	0.994	0.996	0.997
	0.935	0.947	0.961	0.974	0.983	0.988	0.992	0.995	0.997	0.998	0.998
	0.940	0.954	0.968	0.980	0.987	0.992	0.995	0.997	0.998	0.999	0.999
14.3	0.945	0.961	0.975	0.985	0.991	0.995	0.997	0.998	0.999	0.999	1,000
	0.950	0.967	0.980	0.989	0.994	0.997	0.998	0.999	0.999	1,000	1,000
	0.943	0.944	0.958	0.971	0.980	0.987	0.991	0.994	0.996	0.997	0.998
	0.947	0.951	0.965	0.977	0.985	0.990	0.994	0.996	0.997	0.998	0.999
	0.951	0.957	0.971	0.982	0.989	0.993	0.996	0.998	0.998	0.999	0.999
	0.955	0.963	0.977	0.987	0.992	0.996	0.997	0.999	0.999	1,000	1,000
12.5	0.959	0.969	0.982	0.990	0.995	0.997	0.999	0.999	1,000	1,000	1,000
	0.963	0.975	0.986	0.993	0.997	0.998	0.999	1,000	1,000	1,000	1,000
	0.967	0.980	0.990	0.995	0.998	0.999	1,000	1,000	1,000	1,000	1,000
	0.958	0.958	0.972	0.983	0.990	0.994	0.996	0.998	0.999	0.999	0.999
	0.960	0.962	0.976	0.986	0.992	0.995	0.997	0.998	0.999	0.999	1,000
12.5	0.962	0.965	0.979	0.988	0.993	0.996	0.998	0.999	0.999	1,000	1,000
	0.964	0.969	0.982	0.990	0.995	0.997	0.998	0.999	1,000	1,000	1,000
	0.966	0.972	0.984	0.992	0.996	0.998	0.999	0.999	1,000	1,000	1,000

	0.968	0.975	0.987	0.993	0.997	0.998	0.999	1,000	1,000	1,000	1,000
	0.970	0.978	0.989	0.995	0.998	0.999	0.999	1,000	1,000	1,000	1,000

The first column of the Table 1 at the tolerance of 50% specifies values of index  $P_c$  for the subsystems with the n-tuple redundancy. The rest cells of this Table specify values of index  $P_c$  for the subsystems with the aliquant redundancy.

The results, which are presented in the Table 1, demonstrate that for the tolerances of the first level there exist various dependences of criterion of the failure-free operation  $P_c$  from the multiplicity  $K_i$  in the case of changes of these tolerances, as well as changes of probabilities of

components. The nature of these changes can be changed within the wide range: from rapid decrease to rapid increase along with the increase of the multiplicity.

In order to verify stability of this property of the aliquant passive redundancy of various subsystems with due consideration of the tolerances, let us perform similar calculations of the criterion of the failure-free operation  $P_c$  for the tolerances of the second level and present these results in Table 2.

**Table 2:** Values of the failure-free operation  $P_c$  of subsystems depending on  $K_i$ ,  $p$  and tolerances of the second level  $dW_p$

$dW_p$ %	$p$	Redundancy multiplicities $K_i$									
		$K_2$	$K_3$	$K_4$	$K_5$	$K_6$	$K_7$	$K_8$	$K_9$	$K_{10}$	$K_{11}$
40	0.610	0.700	0.658	0.640	0.632	0.626	0.622	0.620	0.619	0.618	0.618
	0.650	0.765	0.751	0.755	0.762	0.771	0.780	0.789	0.798	0.806	0.815
	0.690	0.823	0.832	0.849	0.866	0.881	0.895	0.907	0.917	0.926	0.934
	0.730	0.874	0.896	0.918	0.936	0.950	0.960	0.969	0.975	0.980	0.984
	0.770	0.916	0.943	0.963	0.975	0.984	0.989	0.993	0.995	0.997	0.998
	0.810	0.949	0.973	0.986	0.993	0.996	0.998	0.999	0.999	0.999	0.999
	0.850	0.973	0.990	0.996	0.999	0.999	0.999	0.999	0.999	0.999	0.999
28.6	0.691	0.746	0.727	0.726	0.730	0.736	0.742	0.749	0.756	0.763	0.769
	0.725	0.816	0.824	0.840	0.857	0.872	0.885	0.897	0.908	0.918	0.926
	0.750	0.869	0.891	0.913	0.931	0.945	0.957	0.965	0.972	0.978	0.982
	0.775	0.913	0.941	0.961	0.974	0.983	0.988	0.992	0.995	0.996	0.998
	0.800	0.949	0.973	0.986	0.993	0.996	0.998	0.999	0.999	1,000	1,000
	0.825	0.974	0.991	0.997	0.999	1,000	1,000	1,000	1,000	1,000	1,000
	0.850	0.990	0.998	1,000	1,000	1,000	1,000	1,000	1,000	1,000	1,000
22.2	0.796	0.844	0.860	0.881	0.900	0.915	0.928	0.939	0.948	0.956	0.963
	0.815	0.880	0.904	0.927	0.944	0.957	0.967	0.975	0.980	0.985	0.988
	0.830	0.917	0.944	0.964	0.976	0.985	0.990	0.993	0.996	0.997	0.998
	0.845	0.947	0.972	0.985	0.992	0.996	0.998	0.999	0.999	1,000	1,000
	0.860	0.970	0.988	0.996	0.998	0.999	1,000	1,000	1,000	1,000	1,000

	0.875	0.986	0.997	0.999	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	0.890	0.992	0.998	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
18.2	0.871	0.893	0.919	0.941	0.957	0.969	0.977	0.983	0.988	0.991	0.993
	0.880	0.910	0.938	0.958	0.972	0.981	0.987	0.991	0.994	0.996	0.997
	0.890	0.931	0.958	0.975	0.985	0.991	0.995	0.997	0.998	0.999	0.999
	0.900	0.948	0.973	0.986	0.993	0.996	0.998	0.999	0.999	1.000	1.000
	0.910	0.963	0.984	0.993	0.997	0.999	0.999	1.000	1.000	1.000	1.000
	0.920	0.975	0.991	0.997	0.999	1.000	1.000	1.000	1.000	1.000	1.000
	0.930	0.985	0.996	0.999	1.000	1.000	1.000	1.000	1.000	1.000	1.000

Results of the calculations, which are presented in Table 2, confirm stability of changes of the criterion of the failure-free operation  $P_c$  from multiplenesses  $K_i$  in the case of variations of probabilities of components and variations of tolerances of the second level. Let us present certain results, which are shown in the Tables 1 and 2, in the form of diagrams.

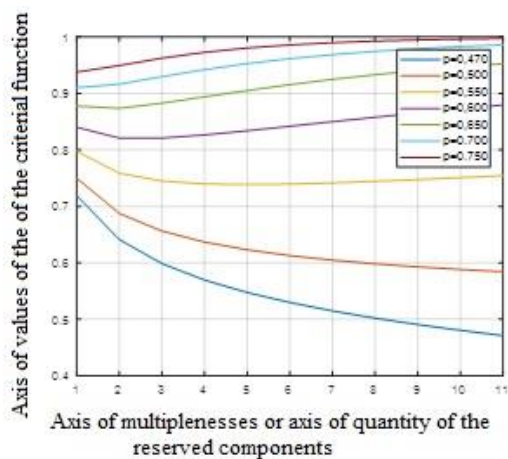


Figure 1: Diagrams of dependences  $P_c$  from the multiplenesses  $K_i$  for various  $p$  at  $dW_p = 50\%$

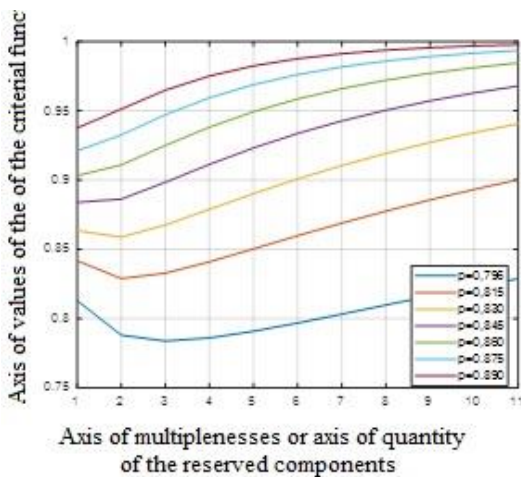


Figure 2: Diagrams of dependences  $P_c$  from the multiplenesses  $K_i$  for various  $p$  at  $dW_p = 25\%$

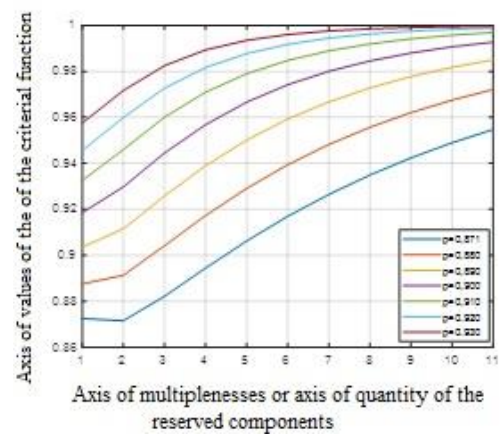


Figure 3: Diagrams of dependences  $P_c$  from the multiplenesses  $K_i$  for various  $p$  at  $dW_p = 20\%$

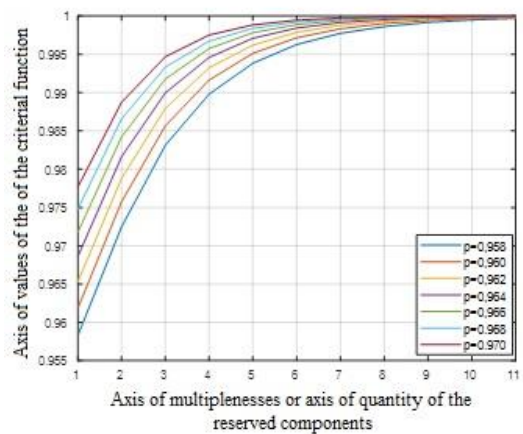
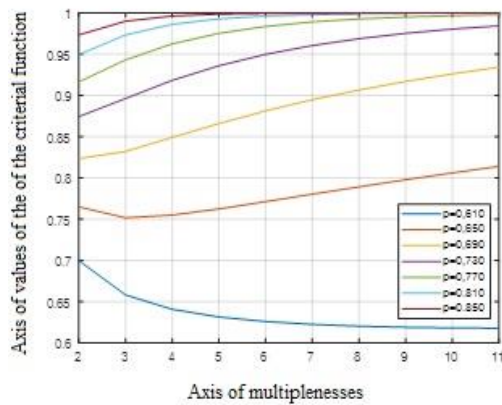
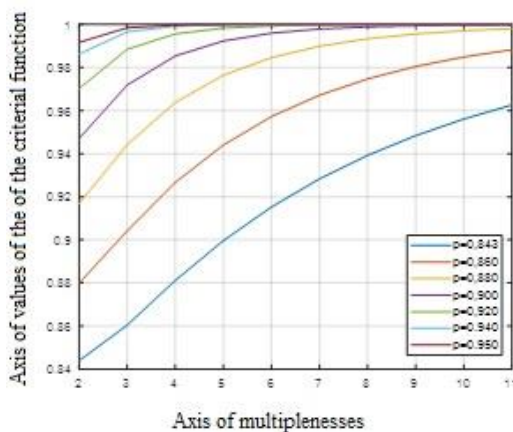


Figure 4: Diagrams of dependences  $P_c$  from the multiplenesses  $K_i$  for various  $p$  at  $dW_p = 12.5\%$

Figures 1, 2, 3, and 4 present certain characteristic dependences of the criterion  $P_c$  from the multiplenesses for the tolerances of the first level, while Figures 5 and 6 present similar dependences for the tolerances of the second level.



**Figure 5:** Diagrams of dependences  $P_c$  from the multiplicities  $K_i$  for various  $p$  at  $dW_p = 40\%$



**Figure 6:** Diagrams of dependences  $P_c$  from the multiplicities  $K_i$  for various  $p$  at  $dW_p = 22.2\%$

The following question is the vital one within this research: "Whether it is always necessary to select a minimum value of the multiplicity of redundancy in order to ensure compliance with the requirement in respect of the failure-free operation of the passive redundant subsystems with due consideration of the tolerances, as it is stated in [7]." In accordance with the results of the calculations that were performed, there is the following decisive answer to this question – "No". Analysis of Tables 1 and 2, as well as analysis of the diagrams, which were presented in Figures 1 – 6 demonstrates that there are no any tolerances, at which it is always useful to select a minimum degree of redundancy of subsystems in order to ensure increase of their failure-free operation [8].

Minimum values of multiplicities ensure only the best values of the failure-free operation indices of the redundant subsystems within certain ranges of probabilities of their components and only in the cases of great tolerances, which exceed certain threshold values. We will refer these threshold quantities as the equilibrium values of tolerances. Deviation of tolerances from the equilibrium values causes changes in the pattern of behaviour of  $P_c$  depending on the multiplicities.

As concerns tolerances of the first level, the equilibrium value of the tolerance is equal to 20%, while for the tolerance of the second level it is equal to 22.2%. If tolerances do not exceed equilibrium values, then minimum multiplicities of redundancy ensure the worst indices of the failure-free operation at any probabilities of components of the redundant subsystems, which their critical values. If tolerances have greater equilibrium values, there exist the ranges of small probabilities, at which

minimum multiplicities of redundancy ensure the higher indices of the failure-free operation as compared with the great multiplicities.

For example, at tolerance  $dW_p = 50\%$  minimum degree of redundancy ensures the best indices of the failure-free operation at probabilities of components of the redundant subsystems  $p_{kr} < p < 0,550$  ( $p_{kr} = 0,470$ ), while at the probabilities  $p > 0,550$  the higher values of multiplicities ensure the higher indices of the failure-free operation. Similarly, if tolerance  $dW_p = 33.3\%$ , then minimum degree of redundancy ensures the best indices of the failure-free operation at probabilities of components of the redundant subsystems  $p_{kr} < p < 0,780$  ( $p_{kr} = 0,691$ ), while at probabilities  $p > 0,780$  the higher values of multiplicities ensure the higher indices of the failure-free operation, and so on

Therefore, it is useful to apply minimum values of multiplicities of redundancy of subsystems at the great tolerances along with not so high requirements to the failure-free operation of the passive redundant subsystems with due consideration of the tolerances in the cases of utilisation of the not very reliable components in the structures of redundancy.

## 6. CONCLUSIONS

This paper describes formation of the model for estimation of the failure-free operation of the passive redundant subsystems with due consideration of tolerances. It is also describes influence of the essential factors of this model upon the accepted index of the failure-free operation. The following factors of influence were investigated: influence upon the probability of failure-free operation during performance of the task of the redundancy structure, influence of the realizable tolerance upon the change of the output parameter, as well as upon the probabilities of failure-free operation of components of the redundant subsystem. The degree of redundancy has been used as the parameter, which determines the redundancy structure. These investigations have demonstrated that there are great possibilities for increase of the failure-free operation of the passive redundant subsystems with due consideration of the tolerances.

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