at stress concentration zones. This would not be visible to the naked eye, although present. Such damage could be at un-acceptable levels but usually not monitored. Most of the times, the bumper assembly is replaced, and vehicle put back to use.

Carrying out a detailed non-linear FEA study to compute damage for anisotropic materials is time consuming and complex due to numerical convergence and other practical issues such as material data availability. Hence development and use of approximate methods is the need of the hour to accelerate the design cycles. Such approximate methods have been developed and successfully deployed for metallic designs [6,7]. Incase of anisotropic materials, few studies have been conducted mostly on metals which exhibit anisotropy that creeps in due to the manufacturing methods [8].

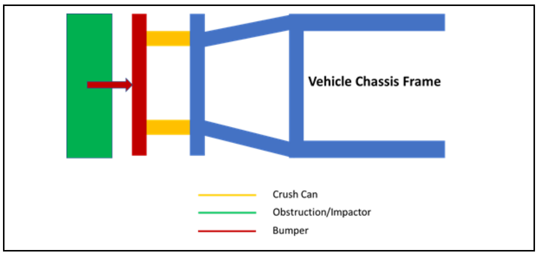

Figure 1: Schematic of CC Design in a Vehicle

1.3 Objective

The objective of the study is as follows:

1. Survey of popular methods available for estimation of plastic strains from Linear FEM methods for isotropic materials.
2. Propose/ Develop Pseudo methods for composites/anisotropic materials to be able to compute the damage evolution (Strain based) based on linear FEM calculations
3. Validate and compare the results from Pseudo methods (Less time Consuming) with those of nonlinear FEM methods (More time Consuming)

1.4 Study relevance

As described earlier, the design focus so far has been on the design of CC which can absorb energy in a sacrificial manner. But in a vehicle, each system or component is expected to absorb certain percentage or fraction of impact energy during an accident. Service loads or slow speed impacts can give rise to localized failures/strains at locations like notches (Stress Concentration zones). These localized failures would reduce the ability of the component to absorb the rated/expected energy during the fatal impact. In such a scenario, even when the safety systems like airbag are deployed, the chances of injury are more, since the CC is impaired to absorb the rated energy due to accumulated damage. These services load and slow speed impacts are quasi-static in nature.

In Practicality most of the designers compare different designs for their ability to absorb energy in a macro manner. This means, there could be different designs with same absorption capacity but the extent of stress concentration at notch could be different for both. The design with lower value of stress concentration at notch should be selected. Otherwise the damage could get accumulated locally at notches during slow impacts which is not visible to the eye.

The Pseudo methods can be used to predict the fraction of the damage accrued due to a service load or slow impact. This way different design/notch configurations can be quickly compared using linear FEM results during initial and/or intermediate stages of design. Also the Psuedo calculations which are simple can be embedded/programmed into safety algorithms which actively keep track of the impact loads (Digital Twin Concept). This way an active damage monitoring system can be put in place and if the system finds the local damage exceeding certain threshold value (Say 10-15%) of damage, it can trigger an alarm to replace the CC with new ones.

1.5 Anisotropic/Orthotropic Materials

Isotropic materials are those which have same material properties in all the directions. Ani-Isotropic materials have different material properties in different directions. Orthotropic elements have different properties along the material axes directions. When analyzing such Structures with such materials the Physical material properties need to be defined along the three material axes.

2. MATERIAL NON-LINEAR ANALYSIS USING FEA METHODOLOGY

Few assumptions are made in linear Stricture analyses/calcualtions using FEA (Finite Element Analysis) Methodology, the two primary ones being the stress/strain relationship and the deformation behavior. The stress is assumed to be directly proportional to strain and the structural deformations are proportional to the loads.

At times the stress at a location in FE model shall be more than the yield, for the applied load. Under such a situation, the stress strain behavior of the system shall be no longer linear, and the solution must follow the stress strain curve of the material. Under such situation, the load is applied in few incremental steps, instead of a single step. At the end of each step, the solver checks for the plastic strains while trying to follow the material stress strain curve. In Such a situation the stiffness matrix at the end of a step shall be input for the next step.

This means, initial conditions at the start of each increment is the state of the model at the end of the previous one. This dependency provides a convenient method for following complex loading histories, such as post Yield behavior. At each increment, the solver iterates for equilibrium using a numerical technique such as the Newton-Raphson method. Due to the iterative nature of the calculations, non-linear FEA is computationally expensive, but reflects the real-life conditions more accurately than linear analyses. The big challenge is to provide a convergent solution at minimum cost (the minimum number of increments).

In a linear Static structural analysis, the following equations are solved:

[K] * [X] = [F]

where [K] is the stiffness matrix, [X] is the displacement matrix or it is sometimes referred to as the unknown's matrix and [F] is the force matrix. With the help of information about nodes, physical and material properties the stiffness matrix for individual elements is calculated and assembled to form the global stiffness matrix.

Then the known displacements are substituted in the displacement matrix. Also, the force matrix is also updated as per the loading conditions. This form a complete set of simultaneous equations which upon solving give out the displacements, strains and stresses at each node as results in a typical commercial FEA software like ANSYS.

For a Nonlinear structural analysis, The Stiffness Matrix is given by [K] = f {X}. This means as the load is being applied in increments, the stiffness matrix varies as a function of the deformation. This is because with the increase of load the stress strain relationship is nonlinear as can be obtained from the stress strain curve. This means for a material non-linear FEA study,

$$f\{X_i\}. * [X_i] = [F_i]$$

Where I = 1 to **n** and $F_1 + F_2 + F_3 + \ldots\ldots\ldots\ldots + F_n = F$. Here the load is applied in **n** number of Increments. Figure 2 and Figure 3 represent the same pictorially.

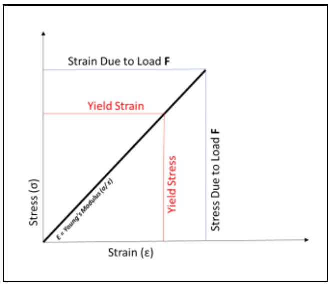

Figure 2: Linear FEA Methodology

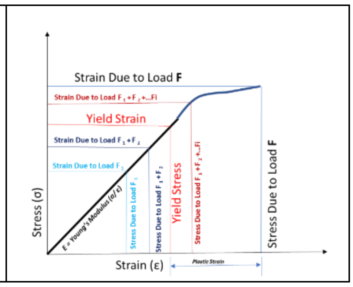

Figure 3: Non-Linear FEA Methodology

2.1 Failure Criteria for Composites

For determining the failure of components that are made of isotropic materials, subjected loading the allowable stress is compared to that of equivalent stress. The equivalent stress is computed based on any one of the theories like Maximum Principals Stress Theory, Maximum Shear Stress Theory (Tresca) and Distortion Energy theory (Von-Mises). However, considering the interaction between different components of stress tensors and also the correlation between calculations and experiments, von-Mises stress criteria has been accepted as the most suitable one.

Compressive load in the form of linear compression has been applied along the axis of the tube. To capture damage, it has been ensured that the applied compression is little higher than that is required to initiate damage process. In case of non-linear analysis for the actual strain calculation, Maximum Stress criteria has been used for damage initiation. For damage evolution, Hashin criteria has been adopted, as the stress components are expected to interact upon initiation of damage [1,16]. The Study for the laminate angle was limited to 45° only as studies have shown for most of the composites the interactive stress based on Damage initiation criteria gradually increases with lamination angle near 40°± and drops thereafter [17].

5.1 Geometry Selections and Load Application for the Study

Several studies have been made on tubular geometries for energy absorption from which it was decided that either a hexagonal or circular tube are capable of absorbing more energy [14]. To study the Effects of Position and Cut-Out Lengths for the notches, the geometry with the most promising energy absorbing geometry has been considered for the study [4, 18-23]. An axial compression that is just enough to create an effective stress that is slightly more than the allowable (Factor of safety less than 1) has been applied.

Figure 10, shows the schematic dimensions of the tube geometry considered and Figure 11, shows the Model and FE mesh considered for the studies.

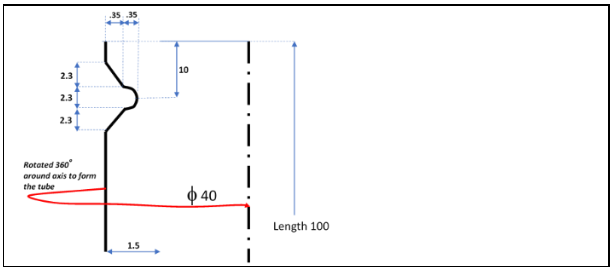

Figure 10: Schematic Dimensions of the tube considered for Study

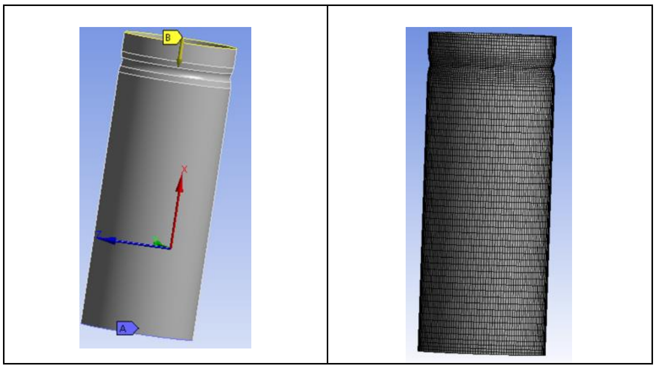

Figure 11: Model and FE mesh considered for the Study

5.2 Carbon-epoxy composite

The following are the material properties (Derived from tests as per ASTM) used for the calculations [12].

Elastic Material Properties (Anisotropic)								
Young's Modulus			***Shear Modulus***			***Poison Ratio***		
E_{11}(MPa)	E_{22}(MPa)	E_{33}(MPa)	G_{12}(MPa)	G_{23}(MPa)	G_{31}(MPa)	V_{12}	V_{23}	V_{31}
58093	58093	9759	3545	2564	2564	0.0154	0.5356	0.1575

Failure Material Properties *(Anisotropic)*		
Description	***Symbol***	***Value (MPa)***
Maximum Allowable Tensile Stress in X Direction	X_T	650
Maximum Allowable Compressive Stress in X Direction	X_C	555
Maximum Allowable Tensile Stress in Y Direction	Y_T	650
Maximum Allowable Compressive Stress in Y Direction	Y_C	555
Maximum Allowable Tensile Stress in Z Direction	Z_T	10
Maximum Allowable Compressive Stress in Z Direction	Z_C	500
Maximum Allowable shear in XY Direction	τ_{12}	181
Maximum Allowable shear in YZ Direction	τ_{23}	132
Maximum Allowable shear in ZX Direction	τ_{31}	132
Thickness of the Tube: 15 mm		

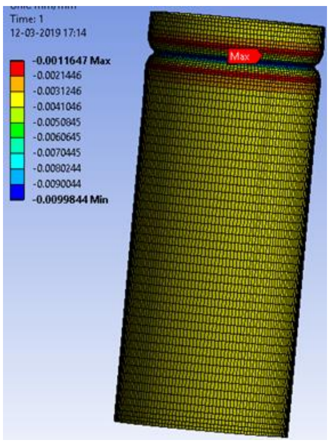

Figure 12: The plot of Total Strain based on Linear FEA Studies

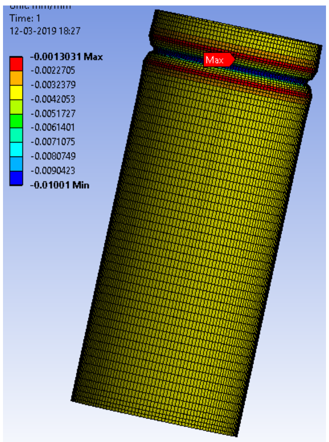

Figure 13: The plot of Total strain based on non-Linear FEA studies

5.3 Carbon–epoxy Laminate

The following are the material properties (Derived from tests as per ASTM) of the Lamina used for the calculations [15].

Elastic Material Properties (Anisotropic) **(GPa)**					***Failure Material Properties (Anisotropic) (MPa)***				
$E_{11} = E_{22}$	E_{33}	G_{12}	$G_{13} = G_{23}$	**Poisson Ratio** V_{12}	X_T	X_C	Y_T	Y_C	τ_{12}
68.9	60	4.06	4.06	0.056	573	693	573	693	94.1
Thickness of Each Layer = 0.1 mm Number of Layers = 15									

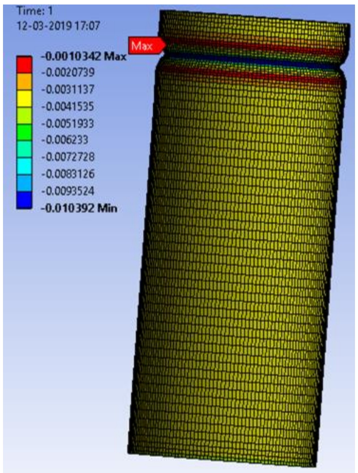

Figure 14: The plot of Total Strain based on Linear FEA Studies for 0 – Deg Lamina

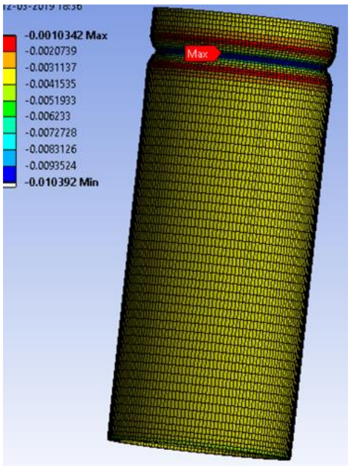

Figure 15: The plot of Total strain based on non-Linear FEA studies for 0 – Deg Lamina

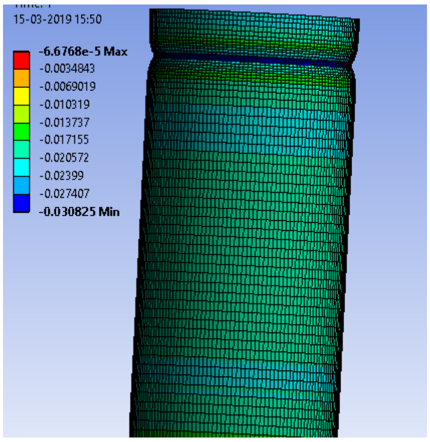

Figure 16: The plot of Total Strain based on Linear FEA Studies for 45– Deg Lamina

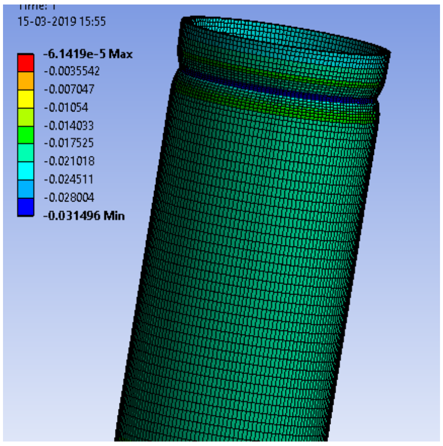

Figure 17: The plot of Total strain based on non-Linear FEA studies for 45 – Deg Lamina

5.4 Results at a Glance

S.No	**Material**	**Maximum Strain as per nonlinear FEA**	**Maximum Strain as per ESED Rule**	**Maximum Strain as per Neuber Rule**	**Maximum Strain as per Arc length Rule**
1	Composite	0.010010	0.009999	0.0104881	**0.020453074**
2	Laminate (Zero Angle)	0.010392	0.010355	0.0108119	**0.022448021**
3	Laminate (45 Deg Angle)	0.031496	0.030862	0.0323661	**0.041208488**

5.5 Discussion on Results

As can be seen from the results the following are the observations:

- As expected, the highest effective stress and strain are noticed at the notch and notch is a stress concentration zone [24-29]
- The Stress/Strain at the notches are highly localized and are several times higher compared to those in the immediate zones
- If the material direction and load direction are same, Maximum Stress/Strain theory can be used for damage initiation identification
- When the material/fiber direction and the load direction are not same, Interactive theory (Such as Hill, Hashin and Puck) need to be used for damage initiation identification
- Irrespective of the material direction, for Damage evolution studies, Interactive theory should be used
- Incase of Laminates, the ply angle has significant effect on the value of interactive stress required for damage initiation. This is because the orientations of the load and material could be different [30-33].
- Change in ply angle for the same load, leads to development of shear stresses making the material/component less stiff
- The values of strain predicted based on Arc Length rule are way off from the actual values
- The values of strain predicted based on ESED method are close to the actual values, but on a lower side [34].
- The values of strain predicted based on Neuber rule are close but on higher side compared to those of actual values

6. CONCLUSIONS

- As can be seen from the study, the Pseudo methods can be successfully used to predict the damage evolution in composite materials in a conservative manner.
- In the absence of full material data such as post yield/Damage-Initiation (Tangent Modulus, Strain rate dependency etc..) and limiting values of Energy for Damage evolution, Pseudo

methods can be still employed to predict the extent of damage evolution (strain) which can be compared against the % elongation of the material, to assess the extent of damage.

- Neuber rule can be safely used to estimate the damage evolution which is localized at the notch area for composites during early and intermediate stages of design, which would be iterative.
- The Pseudo methods can save lot of time there by accelerat the design process cycle, as these methods require only one linear FEM calculations compared to nonlinear FEM calculations which are iterative and at least 5-10 times more expensive computationally, besides requiring lot of material parameters which may not be readily available.
- These methods shall be very helpful during the initial phases of design life cycle to make quick comparisons between multiple notch configurations and to make quick decisions.
- These methods also shall be very handy to compare and choose from multiple notch configurations with different materials, in the absence of readily available limiting strain energy data, which is the practical case.

7. FUTURE SCOPE

7.1 Low Cycle Fatigue

When external loads are applied to a component with a notch, the material response of the notch can be best studied using strain phenomenon rather than stress. The durability of such a member or component can be best estimated from the fatigue life data of the un-notched specimens, that are made of same material and subjected to same strain, through strain control tests. The fatigue life based on initiation and formation of small micro cracks can be studied with good accuracy based on the local strain and time history at the notch in the component, As per Massing's Hypothesis from the stabilized hysteresis loop. These approximate methods discussed should be able to predict the plastic strains for composites with a higher percentage elongation as can be used for equipment like compressor blades.

7.2 GLOSS Method

The GLOSS method described in literature shall be able to predict the damage evolution and/or plastic strains more closely. But it requires 2 linear analysis calculations.

CONFLICT OF INTEREST

The authors declare that they have no conflict of interest.

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